Real Coded Genetic Algorithm for the Design of Digital Differentiator

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Abstract: **Digital Differentiator (DD) play a very important role to find out and approximate the first, second, or higher-order derivative of a digital signals. In this paper real-coded genetic algorithm (RCGA) is proposed for the design of finite impulse response (FIR) digital differentiator. The intent of this paper is to apply a real-coded genetic algorithm with arithmetic-average-bound-blend (AABBX) crossover and wavelet mutation operator for the design of digital differentiator. The values of the filter coefficients are optimized with RCGA approach to satisfying prescribed specifications. The proposed method is, not only accurate and robust but also optimal in the least-squares sense.**

Keywords: **Digital Differentiator (DD), Finite Impulse Response (FIR) filter, least-squares error, real-coded genetic algorithm (RCGA), McClellan-Parks algorithm.**

I INTRODUCTION

Digital Differentiator (DD) play a very important role to find out and approximate the first, second, or higher-order derivative of a digital signals. DDs are used in several areas such as radar, sonar, image processing, communication systems and speech processing systems. Applications such as radar and underwater acoustic signal processing frequently require the accurate measurement of first, second, or higher-order derivatives of digital signals at mid-range frequencies while maintaining acceptable error levels near the cutoff frequencies with reasonable filter lengths. Different methods have been applied in the past years to design the DDs. Higher order finite impulse response (FIR) DDs are very well designed by the modified McClellan–Parks method [1]. The disadvantage of this method is that it gives large errors.

The difference between designed and the ideal response is high in the case of fifth-order DD designed in [2]. The eigenfilter method is extended in [2] by formulating an error function in quadratic form. The Fourier series method is extended in [3] for the design of DDs. Fifth order differentiator have been designed in by Sunder and Ramachandran [4] by modifying the least-squares approach. Higher order DDs have been efficiently designed using WLS technique by Bhosle et al. [6]. Rahenkamp and Vijaya Kumar have modified the program for designing higher order differentiators [7]. These higher-order digital differentiators are very useful for calculation of geometric moments [8] and for biological signal processing [9]. In this paper real real-coded genetic algorithm (RCGA) is proposed for the efficient design of FIR digital differentiator.

The paper is structured as follows. Section 2 describes the FIR digital differentiator design problem statement with four different cases. In Section 3, an error function minimization criterion is described. The real-coded genetic algorithm (RCGA) for designing the optimal digital FIR differentiator is described in Section 4. Section 5 provides the design examples for the even and odd order DDs and Finally, the conclusions and discussions are outlined in Section 6.

II FIR DIGITAL DIFFERENTIATOR

The transfer function of FIR filter is given as:

$$H(z) = \sum_{n=0}^{N-1} h(n)\, z^{-n} \tag{1}$$

where the coefficients *h (n)* (for *n = 1.2, N - 1***)** represent the impulse response of finite length and *N* represents the length of the filter.

The frequency response of the FIR filter is obtained by substituting the frequency variable $e^{j\omega}$ for the z variable, $z = e^{j\omega}$ in Eq. (1)

$$H(e^{jw}) = \sum_{n=0}^{N-1} h(n) e^{-jwn} \tag{2}$$

The corresponding frequency response is characterized by

$$H(e^{jw}) = M(\omega)e^{-j\omega(N-1)/2} \text{ for symmetric filter} \tag{3}$$

$$H(e^{jw}) = M(\omega)e^{j(\pi/2-\omega(N-1)/2)} \text{ for antisymmetric filters} \tag{4}$$

where $M(\omega)$ is the real valued amplitude response given by

$$M(\omega) = \sum_{n=n_0}^{M} b(n)\,p(\omega) \tag{5}$$

Table 1. Different types of FIR filters

Type	$p(\omega)$	M	n_0	N	*Response*	*b(n)*
I	$\cos n\omega$	$(N-1)/2$	0	odd	Symmetric $h(n)=h(N-1-n)$	$2h[(N-1)/2-n]$ $for 1 \le n \le (N-1)/2 and$ $h[(N-1)/2]\ for n=0$
II	$\cos(n-1/2)\omega$	$N/2$	1	even	Symmetric $h(n)=h(N-1-n)$	$2h(N/2-n)$ $for 1 \le n \le N/2$
III	$\sin n\omega$	$(N-1)/2$	1	odd	Antisymmetric $h(n)=-h(N-1-n)$ $h[(N-1)/2]=0$	$2h[(N-1)/2-n]$ $for 1 \le n \le (N-1)/2$
IV	$\sin(n-1/2)\omega$	$N/2$	1	even	Antisymmetric $h(n)=-h(N-1-n)$	$2h(N/2-n)$ $for 1 \le n \le N/2$

Table 1 shows classification of filters into four different fundamental types based on nature of symmetry and its length. The table also gives the conditions of symmetry, expressions for filter coefficient *b(n)*, n_0, *M* and the trigonometric function *p(ω)*.

An ideal *k*th-order differentiator has the following frequency response:

$$H(e^{j\omega}) = D(\omega)\,e^{jk\pi/2} \tag{6}$$

where

$$D(e^{j\omega}) = \left(\frac{\omega}{2\pi}\right)^{k} \qquad 0 \le \omega \le \omega_p \le \pi \tag{7}$$

is the ideal amplitude response of differentiator and ω_p is the highest frequency for which differentiating action is required. For even order digital differentiator i.e. k is even, and then $H(e^{jw})$ is real valued function. So, only a FIR filter with symmetric response i.e. types I and II as from table1 can be used for the design of even-order differentiator. Whereas type I is suitable for designing full band or non-full band differentiator, type II is useful only for designing non-full band differentiator due to the inherent zero folding frequency constraint. Similarly, for an odd-order digital differentiator i.e. k is odd, and then $H(e^{jw})$ is purely imaginary function. So for designing odd order differentiator, FIR filter with an antisymmetric impulse response of type III and IV is used. Type III is only suitable for designing the non-full band odd order differentiator due to the inherent zero folding frequency constraints; type IV is suitable for the full band as well as non-full band differentiator.

III ERROR FUNCTION MINIMIZATION

The optimal coefficients can obtained by minimizing the following weighted mean-squared error with respect to the higher-order DD passband and can be expressed as According to [10], the mean square error is given by

$$E_{mse} = \sum_{l=1}^{K} W(\omega_l)\,E_r^2(\omega_l) \tag{8}$$

Where the relative error $E_r(\omega_l)$ at frequency ω_l is given by

$$E_r(\omega_l) = \frac{E_a(\omega_l)}{D(\omega_l)} \tag{9}$$

Where

$$E_a(\omega_l) = M(\omega_l) \quad D(\omega_l) \tag{10}$$

Is the error function, $W(\omega_l)$ is the frequency-dependent weighting function and K is the number of points at which the error function is sampled. In minimizing E_{rmse}, $\frac{\partial E_{rmse}}{\partial b} = 0$ has been set to obtain a system of linear equations

$$Qb = d \tag{11}$$

The shape of the error function is not known a priori and therefore, $W(\omega_l)$ cannot be found analytically. Consequently, an iterative procedure to identify the appropriate weighting function must be followed.

Let $W_k(\omega_l)$ be the weighting function at the kth iteration. Then $W_{k+1}(\omega_l)$ is written as

$$W_{k+1}(\omega_l) = W_k(\omega l)\,\beta_k^{\theta}(\omega_l) \tag{12}$$

where the updating function, $\beta_k^{\theta}(\omega_l)$ is a function of the envelope of the error function.

Where

$$\beta_k(\omega_l) = \left|E_a(\omega l)\right| \tag{13}$$

Normalize the weights by dividing all the values by the largest value. θ values for the even and odd order differentiators are used as recommended in [11].

IV SOLUTION METHODLOGY

In this paper, a RCGA with genetic operators including arithmetic-average-bound-blend (AABBX) crossover [13] and wavelet mutation is applied for optimizing the filter coefficients of differentiator to minimize amplitude response error between ideal and designed differentiator. The arithmetic-average-bound-blend crossover operator combines the arithmetic, average, bound and blend crossover operators. The arithmetic crossover operation produces some children with their parent's features; average crossover manipulates the genes of the selected parents and the minimum and maximum possible values of the genes and bound crossover is capable of moving the offspring near the domain boundary. The offspring such obtained spreads over the domain so that a higher chance of reaching the global optimum can be obtained. The wavelet mutation operation based on wavelet theory [14] is a powerful tool for fine tuning of the genes to search the solution space locally. This property of wavelet mutation operation enhances the searching performance and provides a faster convergence than conventional RCGA.

Algorithm for Real Coded Genetic:

1) Generate initial population strings randomly.
2) Calculate fitness values of population members.
3) Search for solution among the population? If '*yes*' then GOTO Step 8.
4) Using stochastic remainder roulette wheel selection choose highly fit member of population as parents and generate off-springs according to their fitness.
5) Breed new strings by mating current off-springs. Apply AABBX crossover and wavelet mutation operator to introduce variations and generate offsprings.
6) Substitute existing offsprings with new offsprings by applying competition and selection.
7) GOTO Step 3 and repeat.
8) Stop.

V EXAMPLES AND DISCUSSION

Design examples are provided to demonstrate the design of both odd- and even-order non-fullband differentiators.

Example 1. In this example, the non-fullband differentiator of odd order having p=5 and ω_p=0.95π is considered. A linear phase FIR filter of case 4 with length N=18 is used to design the differentiator. Table 1 shows the filter coefficient of differentiator with the proposed method and Fig. (1), shows the response of designed and ideal differentiator.

Example 2. In this example, the non-fullband differentiator of even order having p=4 and ω_p=0.92π is considered. A linear phase FIR filter of case 2 with length N=32 is used to design the differentiator. Table 2 shows the filter coefficient of differentiator with the proposed method. Fig. (2), shows the response of designed and ideal differentiator

In the above examples cut off frequency of 0.92π and 0.95π is taken for the design of digital differentiator. Digital differentiator having lower cutoff frequency and full-band DD can also be designed with the same technique.

Table 1. Filter coefficients of DD of order 5, length N=10

n	***b(n)***
1	0.0091658
2	-0.00610020
3	0.0026998
4	-0.0072132
5	0.0000904

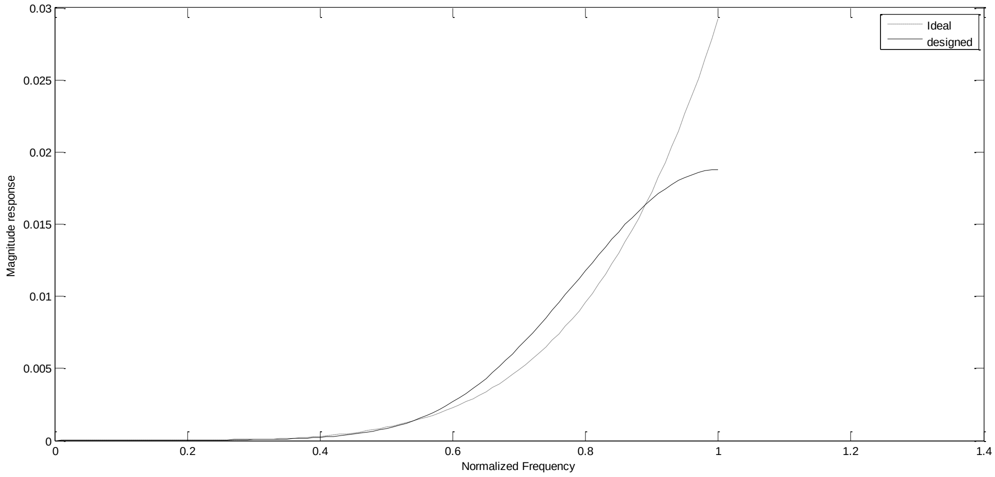

Fig.1. Magnitude response of ideal and designed DD having order p= 5 and length N = 10 with proposed method

Table 2. Filter coefficients of DD of order 4, length N=32.

n	*b(n)*	*n*	*b(n)*
1	0.0029989	9	0.0013100
2	-0.0064977	10	-0.0010112
3	0.0060979	11	0.00075987
4	-0.0045990	12	-0.0005699
5	0.0034895	13	0.00041211
6	-0.0028100	14	-0.00028131
7	0.00209028	15	0.00020111
8	-0.0017102	16	-0.00011120

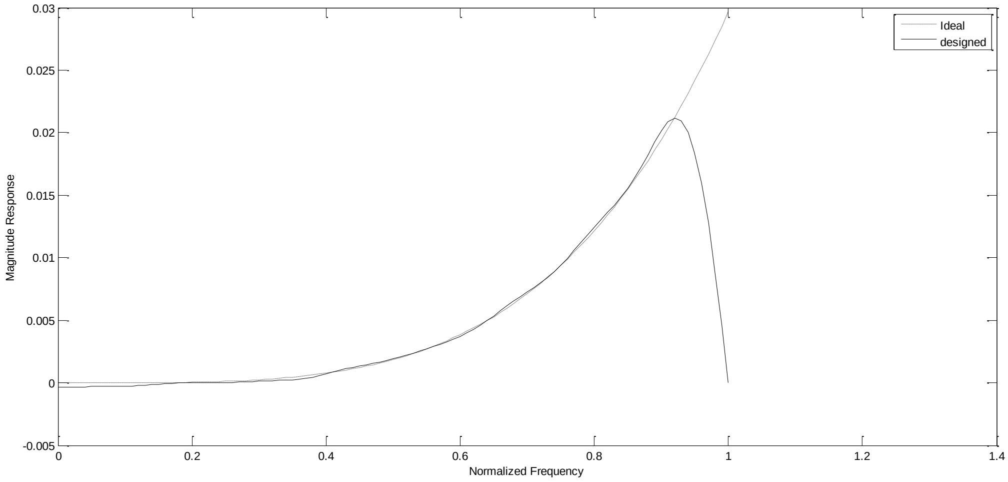

Fig.1. Magnitude response of ideal and actual of DD having order p= 4 and length N = 32 with proposed method.

VI CONCLUSIONS

This paper proposes a RCGA approach for the design of FIR digital differentiator optimization of digital IIR filters considering multiple conflicting objectives. On the basis of results obtained for the design of FIR digital differentiator, it can be concluded that RCGA is a robust algorithm and possesses the capacity for the local tuning of the solutions. RCGA can design a digital differentiator of any even and odd type, while satisfying the prescribed amplitude specifications consistently.

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